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FAILURE ANALYSIS OF COMPOSITES WITH STRESS GRADIENTS

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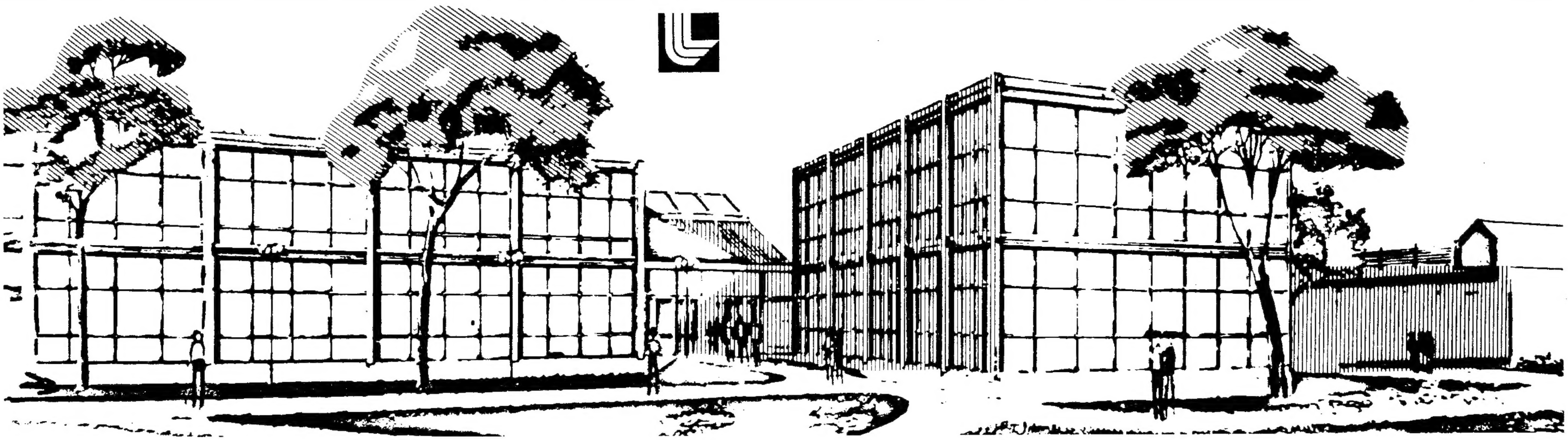

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FAILURE ANALYSIS OF COMPOSITES WITH STRESS GRADIENTS*

Edward M. Wu

Lawrence Livermore Laboratory, University of California
Livermore, California 94550

ABSTRACT

In technical applications of composites, there exists a need to analyze composite strength in terms of stress gradients such as cracks, cutouts, and concentrated loads. Traditional continuum stress analysis cannot fully reconcile the interaction of the stress gradient and the local heterogeneity of fiber matrix and lamination. We present an adaptation of the statistical strength theory to analyze the effect of stress gradients by explicit relation to the intrinsic strength variability of the composite. This method is suitable for failure analysis of composites under a general state of stress without employing an arbitrary averaging parameter.

INTRODUCTION

The development of current composite materials is the result of engineering combination (as distinguished from physical and chemical combination) of several materials in macroscopically multiphase form to achieve certain physical properties not realizable by the constituent materials individually. The resulting composite properties of such multiphase combinations often are anisotropic. These anisotropic physical properties may be partitioned into two categories: (1) those related to averaged global responses (such as stiffness, thermo, conductive, and transport properties), and (2) those related to local phenomena (such as strength, fracture, and interfacial properties). For the first category, continuum analysis and numerical modeling have provided quantitatively accurate predictions. For the second group, a satisfactory reconciliation between micromechanism and continuum analysis has not been made. A most conspicuous departure from the traditional continuum analysis is the current treatment of strength and fracture as isolated phenomena.

The work presented here is an attempt to develop a physical association (which can be statistically quantitative) between strength theories and fracture mechanics. Such a quantitative understanding of the parameters that govern composite failure is imperative to the implementation of fail-safe design and the inspection of critical load-bearing composite structures. Our results also may be useful for predicting the size effect of scaling up laboratory samples to larger size structures in the presence of stress concentrations and stress singularities.

In current research practices, characterization of the strength of anisotropic multiphase composites is usually separated into two broad categories: (1) composite strength in the absence of macroscopic flaws, and (2) composite strength in the presence of macroscopic flaws (and stress risers). These two categories are referred to, respectively, as anisotropic failure criterion characterization and fracture mechanics; usually, they are

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treated as separate physical phenomena. Clearly, such arbitrary categorizing is a consequence of attempts to identify the critical paths of composite strength characterization through association with those experiences gained from isotropic solids. The one-parameter nature of isotropic fracture follows directly from the physical observations that isotropic crack extension is always perpendicular to the direction of maximum tension and that energy dissipation always occurs via a crack-opening mode. Thus, the similarity between mathematical model and physical observation is easily maintained.

In contrast, composites, particularly in the laminated form, exhibit a large range of instability conditions involving various amounts of slow crack growth. In composites, the modes of energy dissipation are not limited to the crack-opening mode; they also include forward sliding and out-of-plane shear. Thus, the crack trajectories seldom follow the maximum tensile stress direction and often lead to nonself-similar crack extension with complex branching. The effects of external loads (symmetric and skew-symmetric to the crack) and combined loading on crack instability need to be documented for composites. Also, the size effect of flaws is far more dominant in composites than in homogeneous isotropic materials.

Whereas the one-dimensional nature of isotropic fracture lends itself to experimental quantification in the form of a single critical stress intensity factor or fracture toughness parameter, the multiple-parameter nature of crack extention in composites precludes empirical permutation of the parameters. For anisotropic composite laminates, there are at least seven primary parameters controlling the fracture characteristics:

(1) Deformational and strength responses of the constituent lamina.
(2) Lamination geometry.
(3) Crack orientation with respect to the material axis of anisotropy.
(4) Crack length.
(5) Nature of the applied stresses.
(6) Energy dissipation associated with the three kinematically admissible modes of crack extension.
(7) Crack trajectory.

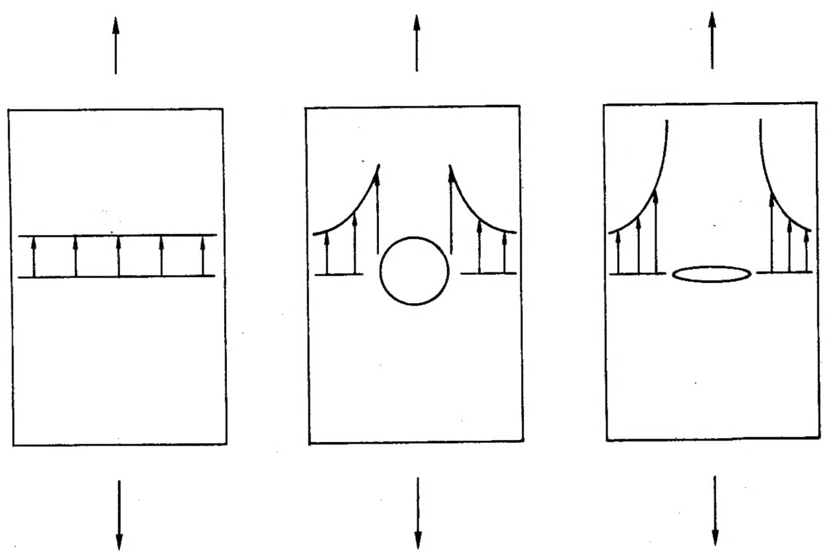
Because of these many parameters, experimental quantification by systematic permutation of the parameters must realistically be viewed as intractable. In this paper, we present an analytical model that reduces the above parameter list from seven to merely the constituent lamina failure criterion and the inherent statistical variability parameter m.

THEORETICAL MODEL

The theoretical model is based on the postulate that:

In the case of qusai-static rupture, the failure of a volume element can be characterized by a weakest link analysis of the local stress.

In particular, the local stress can range from a homogeneous state (as in uniform tension, Fig. 1a), to a stress concentration (Fig. 1b), and finally to a stress singularity in the presence of a crack (Fig. 1c). This postulate provides the bridge between strength theories and fracture mechanics.



(a) (b) (c)

Fig. 1. Range of stress gradients: (a) homogeneous state in uniform tension, (b) stress concentration, and (c) stress singularity in the presence of a crack.

One of the most familiar forms of weakest link characterization of the strength of materials is the Weibull statistic strength theory. The probability of survival P_s, for a material of volume V, and subjected to a spatially dependent stress $\sigma(x_i)$, is represented as:

$$P_s = \exp\left\{ - \int_V \left(\frac{\sigma(x_i) - \sigma_u}{\sigma_o} \right)^m dv \right\} , \qquad (1)$$

where σ_u is the stress threshold below which the probability of failure is zero, σ_o is a normalization parameter, and m is the Weibull parameter that characterizes the variability of observed strength scatter. This Weibull statistical strength theory has been successfully employed in the characterization of brittle ceramics and carbon structures.

Although some *ad hoc* adaptation of this theory to composites has been reported, none has focused on the basic limitations of this Weibull form. From Eq. (1), the implicit assumption is that failure is a one-dimensional process. This implies that identical strength would be observed regardless of whether the material is subjected to uniaxial or complex states of stress. Furthermore, strength properties are assumed to be independent of directions. Generalizations to eliminate these restrictions are needed for a rational characterization of composites. Both of these restrictions can be resolved with a mathematically operational anisotropic failure criterion.

In recent years, numerous failure criterion have been proposed. Examination of their formulations [1] reveals that they are mathematically awkward; some even lack consistency of conversion between stress and strain. Tsai and Wu [2] found that the tensor polynomial failure criterion encompasses maximum flexibility without redundancy and, furthermore, that this criterion lends itself to the design of critical experiments [3]. The tensor polynomial failure criterion is used here, although we emphasize that other experimentally verified criteria may be substituted. The tensor polynomial failure criterion, when expressed in terms of stress, takes the form in contracted notation:

$$f(\sigma_i) = F_i\sigma_i + F_{ij}\sigma_i\sigma_j + F_{ijk}\sigma_i\sigma_j\sigma_k + \dots = 1, \qquad i = 1,2, \dots 6 . \qquad (2)$$

For a typical engineering composite (e.g., graphite/epoxy), the linear and quadratic terms in Eq. (2) provide sufficient correlation of the experimental data as shown in Fig. 1. These experimental data were obtained from tubular samples tested under combined stress conditions along radial loading paths on an axial-rotary-internal pressure mechanical testing machine that is controlled by an on-line digital computer. The experimental details are reported in [4]. The data actually populate a three-dimensional space in $\sigma_1\sigma_2\sigma_6$, but they have been convoluted (or projected) onto the $\sigma_1\sigma_2$ plane for easy comparison. In Fig. 2, the same set of experimental data is convoluted onto the $\sigma_1\sigma_2$ plane by three different failure criteria. Better correlation by the tensor polynomial criterion is exhibited visually and by the lowest RMS (root-mean-square) deviation of experiment from theory.

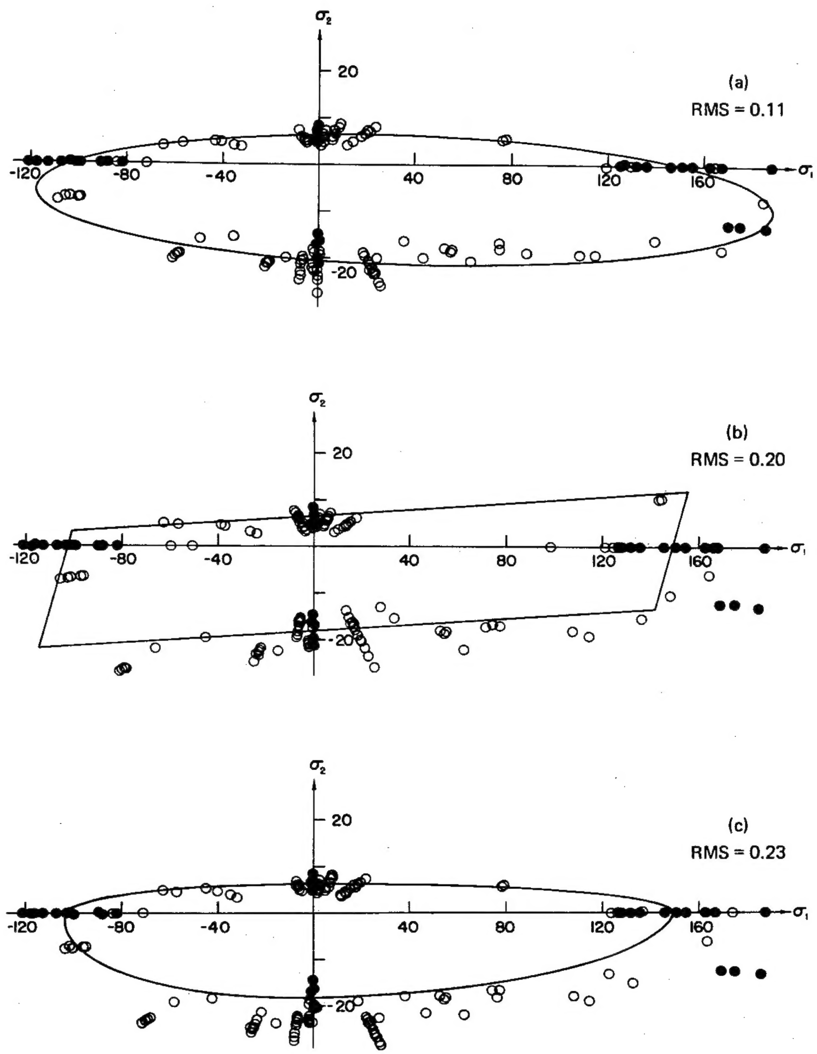


Fig. 2. Failure data of graphite/epoxy lamina convoluted on the $\sigma_1\sigma_2$ plane (root mean square stresses are in ksi): data convoluted by (a) the tensor polynomial failure criterion, (b) the maximum strain failure criterion, and (c) the modified Mises-Hill failure criterion.

The physical interpretation of the failure envelope requires some attention. The composite is assumed to be homogeneous and anisotropic, and to contain a population of randomly distributed microscopic flaws C_1, C_2, ... C_j. Although the flaws are small compared to the characteristic dimension D of the body as depicted in Fig. 3a, continuum analysis reveals that, under arbitrary loads P_i, the state of stress is unbounded at the location of the geometric singularities C_1, C_2, ... C_j, and thus would lead to immediate failure even for extremely small P_i. This is contrary to physical observations. The stresses appearing in Eq. (1) should therefore be interpreted as the average stress acting on a small but finite characteristic volume (specified by a dimension r_c,* Fig. 3a) that fully encapsulates one microscopic flaw. Thus, although the stress is singular inside this characteristic volume r_c, the average stresses external to r_c are bounded and may be used to characterize the failure of this volume through a failure criterion of the form

$$\mathscr{S} \leq \mathscr{F} \quad . \tag{3}$$

Here, $\mathscr{S}$ is the average stress vector acting external to the characteristic volume and is defined in terms of the unit vector $\vec{e}_i$ in the stress of Fig. 3b,

$$\mathscr{S} = \sigma_i \vec{e}_i , \qquad i = 1,2, \ldots 6 \quad . \tag{4}$$

Also, $\mathscr{F}$ is the strength vector to the failure surface $f(\sigma_i)$ as determined by Eq. (1) and as illustrated in Fig. 3b. Under an arbitrary loading P_i, the stress vector $\mathscr{S}$ at any location of the body can be determined through continuum analysis or numerical techniques. It follows that, when criterion $f(\sigma_i)$ is known, the location of a prevalent failure condition can be anticipated by considering the probability of survival of each volume element within the body. For a given volume element V_i (where $V_i > r_c^3$) having a flow density per unit volume ρ subjected to the action of a stress vector $\mathscr{S}$, the probability of survival is

$$P_s = g\left(\frac{\mathscr{F}}{\mathscr{S}}\right)^{\rho V_i} \quad . \tag{5}$$

For the total volume V consisting of V_i volume elements, the cummulative probability of survival is

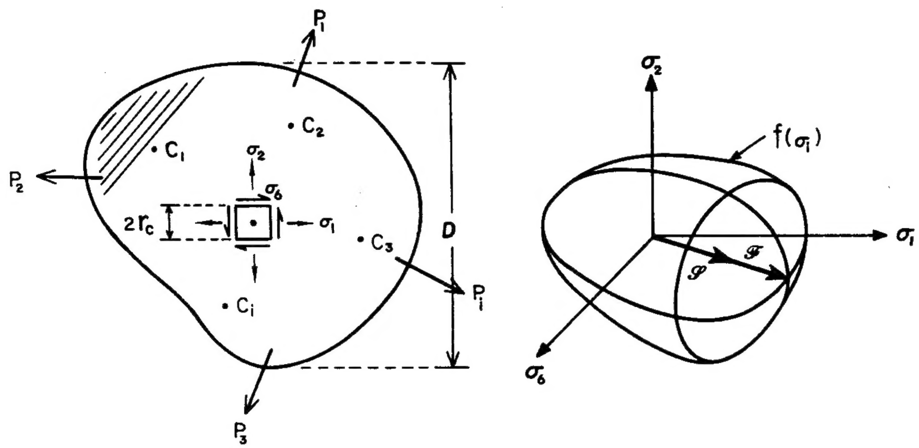
$$P_s = \prod_{i=1}^{n} g\left(\frac{\mathscr{F}}{\mathscr{S}}\right)^{\rho V_i} \quad . \tag{6}$$

Equation (6) can be rewritten in the integral form,

$$P_s = \exp \int_{V_c}^{V} \ln g\left(\frac{\mathscr{F}}{\mathscr{S}}\right) dV \quad , \quad i \frac{\mathscr{F}}{\mathscr{S}} \geq 1 \quad , \tag{7}$$

where the lower limit of integration is the characteristic volume $V_c = 0(r^3)$.

*The explicit determination of this characteristic volume will be discussed after we develop the general form of the statistical failure model.



(a) (b)

Fig. 3. Homogeneous anisotropic body with randomly distributed microscopic flaws in (a) and diagram of the criticality of stress vector $\mathscr{S}$ acting on the characteristic dimension r_c, the failure surface $f(\sigma_i)$, and the strength vector $\mathscr{F}$ in (b).

In this generalized representation of the probability of survival of an anisotropic body, the only restrictions are in the limitation conditions of $g(\mathscr{F}/\mathscr{S})$. To ensure no failure under zero stress,

$$\lim g\left(\frac{\mathscr{F}}{\mathscr{S}}\right) = 1 , \quad \frac{\mathscr{F}}{\mathscr{S}} \to \infty \quad . \tag{8a}$$

To ensure no survival under limiting stress,

$$\lim g\left(\frac{\mathscr{F}}{\mathscr{S}}\right) \to 0 , \quad \frac{\mathscr{F}}{\mathscr{S}} \to 1 \quad . \tag{8b}$$

Any function that satifies Eqs. (8a) and (8b) may be considered as a candidate for characterizing a given composite. In particular, we may choose an exponential form, as did Weibull; i.e.,

$$g\left(\frac{\mathscr{F}}{\mathscr{S}}\right) = \exp - \left(\frac{1}{\frac{\mathscr{F}}{\mathscr{S}} - 1}\right)^m \quad . \tag{9}$$

This leads to an eminently tractable form,

$$P_s = \exp\left\{ - \rho \int_{V_c}^{V} \left(\frac{1}{\frac{\mathscr{F}}{\mathscr{S}} - 1}\right)^m dV \right\} \quad . \tag{10}$$

This particular form is applicable for all ranges of stress distributions ranging from homogeneous state to stress concentration sites. Further simplification is possible where severe stress concentrations (e.g., sharp notches or cracks) cause drastic strength reduction; i.e., $\mathscr{S} << \mathscr{F}$. Under such circumstances, Eq. (10) becomes

$$P_s = \exp\left\{ - \rho \int_{V_c}^{V} \left(\frac{\mathscr{S}}{\mathscr{F}}\right)^m dV \right\} \qquad \text{for } \mathscr{S} << \mathscr{F} \tag{11a}$$

We note that in the one-dimensional case under an applied stress $\mathscr{S} = \sigma$, $\mathscr{F} = X$ where X is the tensile strength, Eq. (11a) reduces to an equation of the Weibull form (Eq. (1)) where the stress threshold σ_u is set to zero:

$$P_s = \exp\left\{ - \rho \int_{V_c}^{V} \left(\frac{\sigma}{X}\right)^m dV \right\} = \exp\left\{ - \int_{V_c}^{V} \left(\frac{\sigma}{\sigma_0}\right)^m dV \right\} . \tag{11b}$$

Hence we see that the Weibull form implies severe stress risers which account for its success in characterizing "brittle" materials.

In our generalization (Eq. 10), we not only introduce the generality of anisotropic strength, we provide for the extended range of application to local stress site from mild stress concentrations to seven stress singularities. In this generalization, we also specify a lower limit of integration in the computation of the cummulative probability of survival (the characteristic volume V_c or the characteristic dimension r_c). This characteristic dimension r_c is the limit of the continuum and may be explicitly determined by exploring the effect of a stress gradient on the probability of survival.

Traditionally, in the deterministic correlation of local point stress to strength, only the stress magnitude is taken into account. As a consequence, the correlation is not able to treat cases where stress becomes singular, e.g., around crack tips and dislocation sites. The stress gradient effects are implicitly taken into account in the Weibull form (Eq. 1). However, in the actual computation involving stress singularity, the integral becomes ill behaved.

We therefore desire to examine explicitly the effect of stress magnitude and stress gradient by a second postulate:

> *For a given material, there exists, at a characteristic stress magnitude $\mathscr{S}_c$, a limiting stress gradient $\mathscr{S}'_c$, above which no stress gradient effect on strength can be measurable.*

We can carry out this exploration using the scalar stress component with no loss of generality; i.e., let $\mathscr{S}_c \rightarrow \sigma_c$ and $\mathscr{S}'_c \rightarrow \sigma'$, where $\sigma' = (d\sigma/dX)$ (see Fig. 4).

For a small (by definition) characteristic dimension r_c, we take the stress gradient to be constant $0 \leq X < r_c$. Hence the stress distribution within r_c is

$$\sigma = \sigma_c + \sigma' X \quad . \tag{12}$$

Because we seek the effect of a severe stress concentration, we can utilize Eqs. (11a) or (11b) to compute the probability of survival of this element within r_c (accounting for stress gradient effect):

$$P_s \Big|_{\mathscr{S}'} = \exp\left\{ -\int_0^{r_c} \left(\frac{\sigma_c + \sigma' X}{\sigma}\right)^m dX \right\} ,$$

$$= \exp\left\{ - \frac{1}{\sigma_0{}^m} \frac{(\sigma_1 + \sigma' r_c)^{m+1} - \sigma_c{}^{m+1}}{\sigma' \ (m+1)} \right\} . \tag{13}$$

The corresponding probability of survival of a homogeneous stress σ within an identical volume is

$$P_s \Big|_{\sigma \text{ homogeneous}} = \exp\left\{ - \int_0^{r_c} \left(\frac{\sigma}{\sigma_0}\right)^m dX \right\} ,$$

$$= \exp\left\{ - \left(\frac{1}{\sigma_0{}^m}\right) \sigma^m r_c \right\} \quad . \tag{14}$$

For equal probability of survival, the ratio of an identical volume, we can equate Eq. (13) to Eq. (14) and obtain

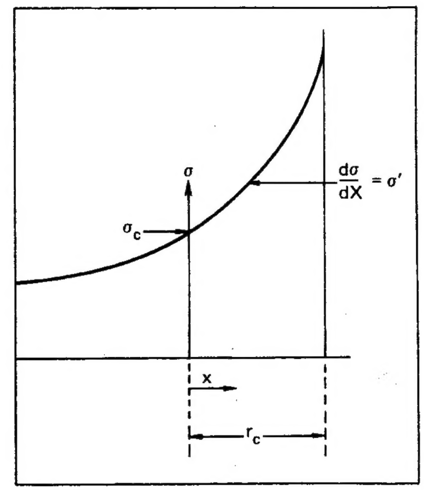


Fig. 4. Plot of the scaler stress component where the characteristic stress magnitude $S_c \rightarrow \sigma_c$, and the limiting stress gradient $S'_c \rightarrow \sigma'$, where $\sigma' = (d\sigma/dX)$, r_c is the characteristic dimension, and X is tensile strength.

$$\frac{\sigma_c}{\sigma} = \left[\frac{(m + 1)\,\frac{\sigma' r_c}{\sigma_c}}{\frac{(1 + \sigma' r_c)^{m + 1}}{\sigma_c} + 1} \right]^{\frac{1}{m}} \quad . \tag{15}$$

We note that the highest stress gradient occurs in the continuum analysis of cracks. Thus, the strength reduction in Eq. (15) will have a lower limit equal to the strength reduction associated with the presence of a crack. For the isotopic case, this strength reduction limit is

$$\frac{\sigma_c}{\sigma} \geq \left(\frac{\frac{k_c}{\sqrt{2r}}}{X} \right) \quad , \tag{16}$$

where k_c is the critical stress intensity factor of the fracture of a crack and X is the highest attainable tensile strength of a uniformly loaded sample. The limiting characteristic dimension r_c for which the limiting stress gradient exists may be determined from Eqs. (15) and (16). For an isotopic crack fracture in a self-similar manner, the stress gradient is

$$\sigma' = \frac{d}{dX} \left. \frac{k_c}{\sqrt{2X}} \right|_{X=r_c} = \frac{-k_c}{2\sqrt{2}}\, r^{-3/2} \quad . \tag{17}$$

Substituting Eq. (17) into Eqs. (15) and (16) yields

$$r_c = \frac{1}{2} \left[\frac{2(1 - \frac{1}{2})^{m + 1}}{(m + 1)} \right]^{2/m} \left(\frac{k_c}{X} \right)^2 \quad . \tag{18}$$

For anisotropic composites, the tensile strength X needs to be replaced by the strength vector $\mathscr{F}$ in the direction of crack extension. With this limiting dimension known, Eqs. (10) or (11) may be used to compute the statistical strength of the composite in the presence of stress gradients. This formulation is not only operationally explicit, it also is physically meaningfull. We note that, according to Eq. (15), the characteristic dimension r_c is related to the strength scatter of the material as characterized by the Weibull parameter m, and that

$$m \to 0 , \quad r_c \to \infty ;$$
$$m \to \infty , \quad r_c \to \frac{1}{2} \left(\frac{k_c}{X} \right)^2 \quad . \tag{19}$$

Because the strength scatter is inversely proportional to the Weibull parameter m, the first limiting condition coincides with the intuitive notion that large scatter and inhomogeneity require a large characteristic volume. The second limiting condition is the case of deterministic strength in which we recover the deterministic formulation of r_c as proposed by Wu [5].

We tested this limiting strength gradient posulate with experimental data on the size effect of circular holes in quasi-isotropic glass/epoxy composites. It has been reported [6] that the strength reduction due to circular holes is dependent on hole size and that, for small hole sizes, the strength reduction is considerably smaller than the theoretically predicted elastic stress concentration of 3. Using the elasticity solution of stress distribution around a circular hole and Eq. (10), we plotted the strength reduction as a function of hole size, together with experimental measurements from Ref. [6] (Fig. 5). For these limited experimental data, we observe an encouraging correlation with the predictions for Weibull parameters between 20 and 30. This agrees well with the literature value of m = 25 for quasi-isotropic fiberglass composites in tension.

CONCLUSION

We have generalized Weibull's statistical strength theory to account for complex states of stress and anisotropic strength so that the failure analysis of composites can include statistical strength and size effects. In addition, we have postulated the existence of a limiting strength dependence on the stress gradient. From this postulate, we have derived a relation that enables the explicit evaluation of a critical dimension which defines the limit of the continuum as a function of material variability. This approach offers a rational link between continuum analysis and local failure sites that is quantitative in terms of established statistical parameters. Tentative but encouraging correlations were observed between Weibull predictions and limited experimental results on strength reduction due to circular holes. Further confirmation of this correlation will require additional experimental results on the stress concentration site with different stress gradients.

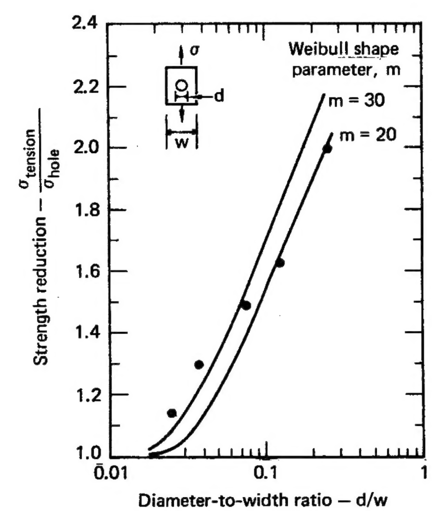


Fig. 5. Plot of strength reduction ($\sigma_{tension}/\sigma_{hole}$) of a quasi-isotropic fiberglass composite with a small hole.

REFERENCES

[1] Wu, E. M. "Phenomenological Anisotropic Failure Criterion" in Composite Materials, (Academic Press, 1974), Vol. 2., Vol. Ed., G. P. Sendeckyj, Series Eds., L. J. Broutman and R. H. Krock.

[2] Tsai, S. W. and Wu, E. M., "A General Theory of Strength for Anisotropic Materials", Journal of Composite Materials, Vol. 5 (1971).

[3] Wu, E. M. "Optimal Experimental Measurement of Anisotropic Failure Tensors" Journal of Composite Materials, Vol. 6 (1971).

[4] Wu, E. M. and Jerina, K. L. "Computer-Aided Mechanical Testing of Composites", Materials Research and Standards, Vol. 12 (1978).

[5] Wu, E. M. "Strength and Fracture of Composites" in Composite Materials (Academic Press, 1974), Vol. Ed., L. J. Broutman, Series Eds., L. J. Broutman and R. H. Krock.

[6] Whitney, J. M. and Nuismer, R. J. "Stress Fracture Criterion for Laminated Composites Containing Stress Concentrations," Journal of Composite Materials, Vol. 8 (1974).

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